THE PROBLEM CORNER 2

As mathematics teachers it is important to continue to practise our art. One important way to do this is by solving non-standard problems. In each of the 2021 MASA newsletters we are offering 5 such questions for you and your interested and motivated students. We invite your solutions and will acknowledge them in the following newsletters. Please accept the challenges, we also invite you to supply a challenging problem for MASA to use in future Problem Corners.

JUNIOR PROBLEMS

J 1

What is the smallest 5-digit number which is divisible by 9 and consists only of even digits?

J 2

Given a number N=523ABC where A, B and C are digits, find the values of A, B and C such that N will be divisible by 7, 8 and 9.

J 3

Integers a, b, c and d are such that $a < 2b$, $b < 3c$ and $c < 4d$ where $d < 101$.
What is the largest possible value that a can have?

J 4

In the subtraction problem below A, B, C and D are digits.
What are their values?

$$\begin{array}{r} 8\,A\,3\,B \\ -\,C\,4\,D\,1 \\ \hline 4\,4\,5\,1 \end{array}$$

J 5

In a bag of marbles, $\frac{4}{7}$ of the marbles are blue and the rest are red.

If the number of red marbles are trebled and the number of blue marbles is increased by 5, the fraction of red marbles is now $\frac{9}{14}$.

How many blue marbles are now in the bag?

SENIOR PROBLEMS

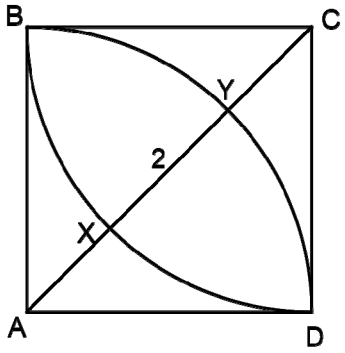
S 1

$\frac{1}{a+b}$, $\frac{1}{b+c}$, $\frac{1}{c+a}$ where a, b and c are integers, are three numbers in an arithmetic sequence.

Prove that c^2, a^2 and b^2 are also in an arithmetic sequence.

S 2

In the square ABCD arcs are drawn with centres at A and C, with radii equal to the side length of the square.
These arcs cut the diagonal AC at X and Y.
If XY=2, what is the area of the square ABCD?



S 3

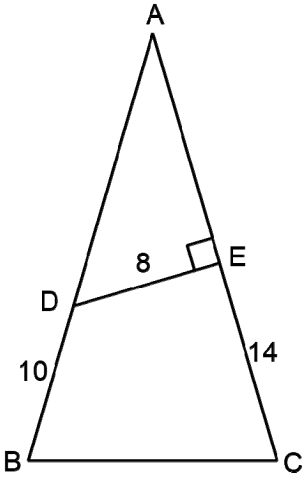
A stack of tennis balls form a pyramid with a square base of 8×8 tennis balls.
Each ball above level one rests in a pocket formed by 4 balls on the level below it.
A single ball completes the stack.

(a) How many tennis balls are in the stack?

(b) If each ball has a radius of 5 cm, how high is the stack?

S 4

In the figure the triangle ABC is isosceles with AB = AC.
D and E are points on AB and AC such that BD = 10, CE = 14,
DE = 8 and the angle DEA is 90°.
What is the area of the triangle ABC?



S 5

Find the smallest positive solution of the equation $\tan 3\theta = \cot 4\theta$.

CHALLENGE PROBLEMS

C 1

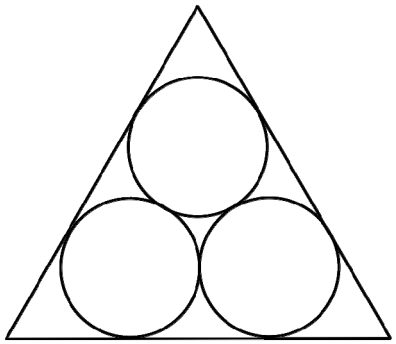
$f(x)$ is a quadratic function. When $f(x)$ is divided by $(x-1)$ the remainder is twice the remainder obtained when $f(x)$ is divided by $(x+2)$.

If $f(0) = 8$ what is the value of $f(5)$?

C 2

Inside an equilateral triangle of side length x are three identical circles of radius 1 unit. The sides of the triangle are tangential to the circles as shown in the diagram.

Find the value of x.



C 3

Find the smallest x for which $[x^2] - [x]^2 = 2016$.

Mathematics can be fun