THE PROBLEM CORNER 1

As mathematics teachers it is important to continue to practise our art. One important way to do this is by solving non-standard problems. In each of the 2021 MASA newsletters we are offering 5 such questions across differing levels for you and your interested and motivated students. We invite your solutions and will acknowledge them in the following newsletters. Please accept the challenges, we also invite you to supply a challenging problem for MASA to use in future Problem Corners.

JUNIOR PROBLEMS

J1

In this addition sum A, B and C are digits, what are their values?

$$\begin{array}{r} A\ B\ B \\ +\ B\ B\ A \\ \hline C\ A\ B\ C \end{array}$$

J2

Alice wrote down all the integers from 21 to 2021

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(a) How many integers did she write down?

(b) How many digits did she write down?

(c) What was the 500th digit she wrote down?

(d) What was the sum of all the numbers she wrote down?

(e) How many of the numbers she wrote down were palindromic numbers?

J3

When asked how many children she had, the mother replied "My youngest has $\frac{1}{2}$ as many sisters as brothers, and my eldest has two more brothers than sisters.

How many girls and boys does the mother have?

J4

How many isosceles triangles with sides of integer length and perimeter 50 units are there?

J5

What is the sum of the first 50 terms of the sequence below?

2, 3, -5, 7, 9, -11, 13, 15, -17, 19,21, -23, ...

SENIOR PROBLEMS

S1

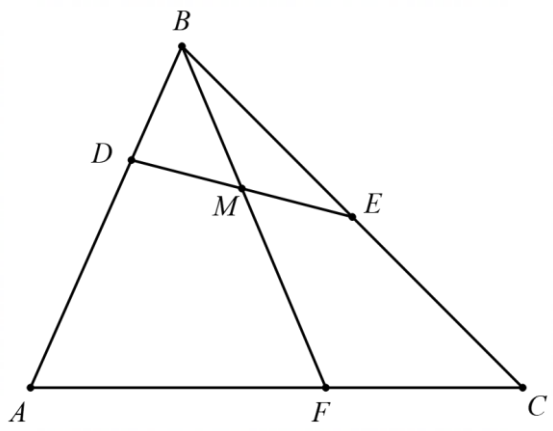
Two positive integers have a sum S, difference D, product P and quotient Q.

What are the integers if S + D + P + Q = 243?

S2

In the figure BE = EC , DM = ME and AD = 2 BD.

Find the ratio AF : FC ?



S3

The radius of the incircle of the right-angled triangle ABC is 13.

Find the possible integer values of the sides a, b and c of the triangle.

S4

Solve the equation $(a + b)^b = a^b + 1413$ where a and b are positive integers.

S5

A quadratic with integer coefficients cannot have a discriminant of 23.

Prove this statement.

CHALLENGE PROBLEMS

C1

The equation $x^2 - 97x + A = 0$ has roots equal to the fourth power of the roots of $x^2 - x + B = 0$. What is the value of A?

C2

A 5-digit number is a multiple of 41. If the highest order digit is removed and placed to the right of the unit's digit, the new number is a perfect cube.
What was the original number?

C3

PTOLEMY'S Theorem fits nicely amongst the circle theorems.
Look it up and use it to solve the following problems.

ABC is an equilateral triangle inscribed in a circle and P is a point on the arc BC.

(a) Prove that $PA = PB + PC$.

(b) Given that PA and BC intersect at D prove that $\frac{1}{PD} = \frac{1}{PB} + \frac{1}{PC}$.

(This last formula is useful in problem solving involving resistances and optics -- check this out as an exercise)

Mathematics can be fun