HOW ANIMALS KEEP THEIR COOL

This unit, and the accompanying unit, “Baby in a car investigation”, explore how mathematics can be used to solve problems and make suggestions for possible project topics.

The Australian Curriculum Problem Solving Proficiency states

- Students develop the ability to make choices, interpret, formulate, model and investigate problem situations, and communicate solutions effectively. Students formulate and solve problems when they use mathematics to represent unfamiliar or meaningful situations, when they design investigations and plan their approaches, when they apply their existing strategies to seek solutions, and when they verify that their answers are reasonable.

This particular unit relates to the Year 10 Content Description:

Using units of measurement

Solve problems involving surface area and volume for a range of prisms, cylinders and composite solids (ACMMG242)

How easy is it for animals to keep warm in winter and cool in summer?

You will probably have noticed how cats curl up in a tight ball in winter and stretch out in summer.

What are they changing when they do this?

They are changing their **exposed surface area**. Heat is lost through the skin – the larger the area exposed the faster is the loss of heat.

On cold days, animals reduce the loss of body heat by curling up into a ball, while on hot days they try to lose heat as quickly as possible by sprawling out.

How can cattle and horses stand outside in the cold weather when a dog, cat, mouse or human would not without experience the bitter cold?

How can we measure how easy it is to keep warm? What do we need to measure? What are the factors that can change?

A pot of hot food will cool slowly in a pot, but much more rapidly when spread out on a plate.

The important factor is not just the surface area. A large animal has a large surface area just because it is large. What matters is the degree of ‘curling up’ or ‘sprawling out’.

For example, which of these is more ‘sprawled out’?

- a 30 x 20 cm piece of paper 0.01 cm thick, or
- a cube with sides 10 cm long?

You will see that they have the same surface area but as the paper is flatter, it is more likely to lose heat more quickly.

What we need to measure is the rate of heat loss in relation to the body size.

This can be measured using the ratio of surface area to volume (i. e. $\frac{Surface\ \ Area}{Volume}$)

Work out the ratio of surface area to volume for the paper and the cube.

Paper Surface area (ignoring the edges) = 2 x 20 x 30
= 120 cm^2

Volume = 20 x 30 x 0.01
= 6 cm^3

$\frac{Surface\ \ Area}{Volume}$ = $\frac{120}{6}$

= 20

Cube Surface area = 6 x 10 x 10
= 600 cm^2

Volume = 10 x 10 x 10
= 1000 cm^3

$\frac{Surface\ \ Area}{Volume}$ = $\frac{600}{1000}$

= 0.6

These results indicate that the lower the value of this ratio, the less heat is lost. The higher values indicate a greater heat loss.

To further explore the rate of heat loss we need to simplify the shape of animals to make it possible to use some mathematical formulae.

We will approximate our animals to a cylinder' where the animals are 2.5 times longer than they are across – their diameter. This would mean that the length is 5 times the radius.

Our formulas will be:

Surface Area (SA) = $2\pi r^2 + 2\pi r(5r)$

= $2\pi r^2 + 10\pi rr$

Volume (V) = $\pi r^2(5r)$

= $5\pi r^3$

	Kitten	Cat	Labrador	Horse	Elephant
r (in cm)					
Surface Area (SA)					
Volume (V)					
$\frac{Surface\ \ Area}{Volume}$ (SA / V)					

Why…

- do elephants and rhinos have big ears and wrinkled skin?
- do smaller animals have more fir than most larger animals?
- do smaller animals tend to be shaped more like a sphere than larger animals?